On the Evolution of the Density PDF in Strongly Self-gravitating Systems

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The time evolution of the density probability distribution function (PDF) is formulated and solved in the free-fall approximation. We demonstrate that a pressure-free collapse results in a power-law tail on the high-density tail of the PDF, with the slope quickly asymptoting to the functional form $\mathrm{PDF}_M(\rho) \propto \rho^{-0.54}$ for the mass-weighted PDF and $\mathrm{PDF}_V(\rho) \propto \rho^{-1.54}$ for the volume-weighted one. Comparison of observed column density PDFs with those derived from our model suggests that observed star-forming cores are roughly in free-fall.

Introduction

The density PDF is a powerful tool for analysing astrophysical systems; in both non-gravitating and strongly self-gravitating systems, it reveals key aspects of the underlying physical processes. For supersonic non-gravitating turbulent gas in an isothermal environment the density PDF is log-normal (Vazquez-Semadeni 1994, Padoan et al. 1997). When self-gravity becomes important, the probability of finding dense regions increases and a power-law tail develops on the high-density side of the PDF (Klessen 2000, Slyz et al. 2005, Collins et al. 2012). The density PDF can be used to evaluate key aspects of star formation, like the efficiency and the stellar initial mass function (IMF) (Krumholz 2005, Padoan et al. 1997, Hennebelle, Chabrier 2008, Federrath, Klessen 2012).

We develop a model for the evolution of the density PDF, based on free-fall collapse. We start with the initial density PDF of a gaseous system, not specifying its spatial structure, nor how it evolved to this state. We then apply the free-fall analysis directly to the density PDF.

Analytic Collapse Model

We start with the free-fall collapse of a sphere that obeys

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2} = -\frac{4\pi G \rho_0 r_0^3}{3r^2}. \qquad (1)$$

Transformation to dimensionless quantities using $\zeta = r/r_0$ and $\tau_{\mathrm{exact}} = t/t_{\mathrm{ff}}$ gives

$$\tau_{\mathrm{exact}} = \frac{2}{\pi}\left(\arccos\sqrt{\zeta} + \sqrt{\zeta(1-\zeta)}\right) \qquad (2)$$

(e.g., Hunter 1962, Tohline 1982). An analytic approximation for $\zeta(\tau)$ can be obtained by setting

$$\tau = \sqrt{1-\zeta^{a/2}}. \qquad (3)$$

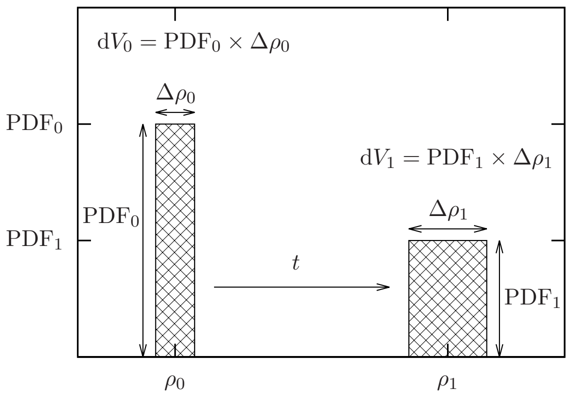
Equation 3 deviates least from the exact solution (equation 2) if we substitute $a = 3.2233$. Assuming the mass in the sphere, $M = 4\pi/3\,\rho(t) r(t)^3$, to be constant during the collapse, equation (3) gives

$$\rho(\tau) = \rho_0 \left(1-\tau^2\right)^{-6/a}. \qquad (4)$$

With this equation, we can now explicitly compute the density as a function of time starting with an initial density ρ_0.

Time Evolution of the PDF

For free-fall from rest, the density is a monotonic function of time, i.e., evolutionary paths of differnent initial densities do not cross. We can use this to derive an analytic description for the time evolution of the density PDF:



The area of a bin on this plot stays constant over time, and so

$$\mathrm{PDF}_V(t_1) = \mathrm{PDF}_V(t_0)\frac{\Delta\rho_0}{\Delta\rho_1}. \qquad (5)$$

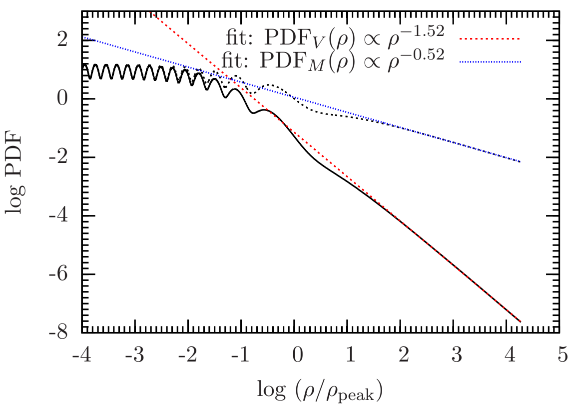
Starting with an arbitrary but fixed $\Delta\rho_0$ at time t_0, the value for $\Delta\rho_1$ at time t_1 can be calculated from the analytic function for $\rho(t)$.

High-density Tail

We now make use of the simple approximation for $\rho(t)$ (equation 3), to derive the time evolution of the density PDF and, in particular, we focus on the high-density part. After some algebra, we end up with

$$\lim_{\rho\to\infty} \frac{\mathrm{d}\log \mathrm{PDF}_V}{\mathrm{d}\log\rho} = -\frac{a}{6} - 1 = -1.54. \qquad (6)$$

Hence, at late times the tail of the PDF has a universal slope, independent of the slope of the initial PDF, provided this is finite.



Variations from Free-fall

Collins et al. (2011) performed simulations of magnetised clouds, finding slopes ranging from $\mu = -1.64$ to -1.80, steeper than in the purely hydrodynamic case; the stronger the magnetic field, the steeper is the tail of the PDF.

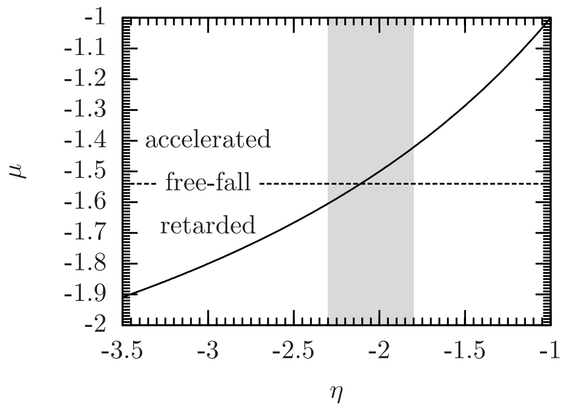
We can understand this behaviour qualitatively by investigating the influence of the parameter a. Although a is introduced as a simple mathematical device, to match the approximate function (equation 3) to the true collapse solution, we can mimic other physical effects by changing its value.

For $a > 3.2233$ (steeper slope), the collapse is delayed initially, but speeds up towards the end. This is qualitatively what happens in marginally unstable density regimes, where the early phase of the collapse is still influenced by stabilising effects like thermal pressure or magnetic fields. Conversely, the tail is shallower if collapse is faster than pure free-fall. This is the case in converging flows, where the collapse does not start with zero velocity, but instead with an initial converging velocity field.

Application to Observational Data

Converting the volume density PDF to a column density PDF ($\mathrm{d}A/\mathrm{d}\Sigma$), we can derive a relation between the slopes,

$$\mu \equiv \frac{\mathrm{d}\log V}{\mathrm{d}\log\rho},\ \eta \equiv \frac{\mathrm{d}\log A}{\mathrm{d}\log\Sigma},\ \mu = \frac{3\eta}{2-\eta}. \qquad (7)$$



Using the slope of the column density PDF, we can determine how close observed structures are to free-fall collapse. The shaded area corresponds to observations by Kainulainen et al. (2009) and Schneider at al. (2012) and suggests that these star-forming cores are collapsing roughly in free-fall.

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