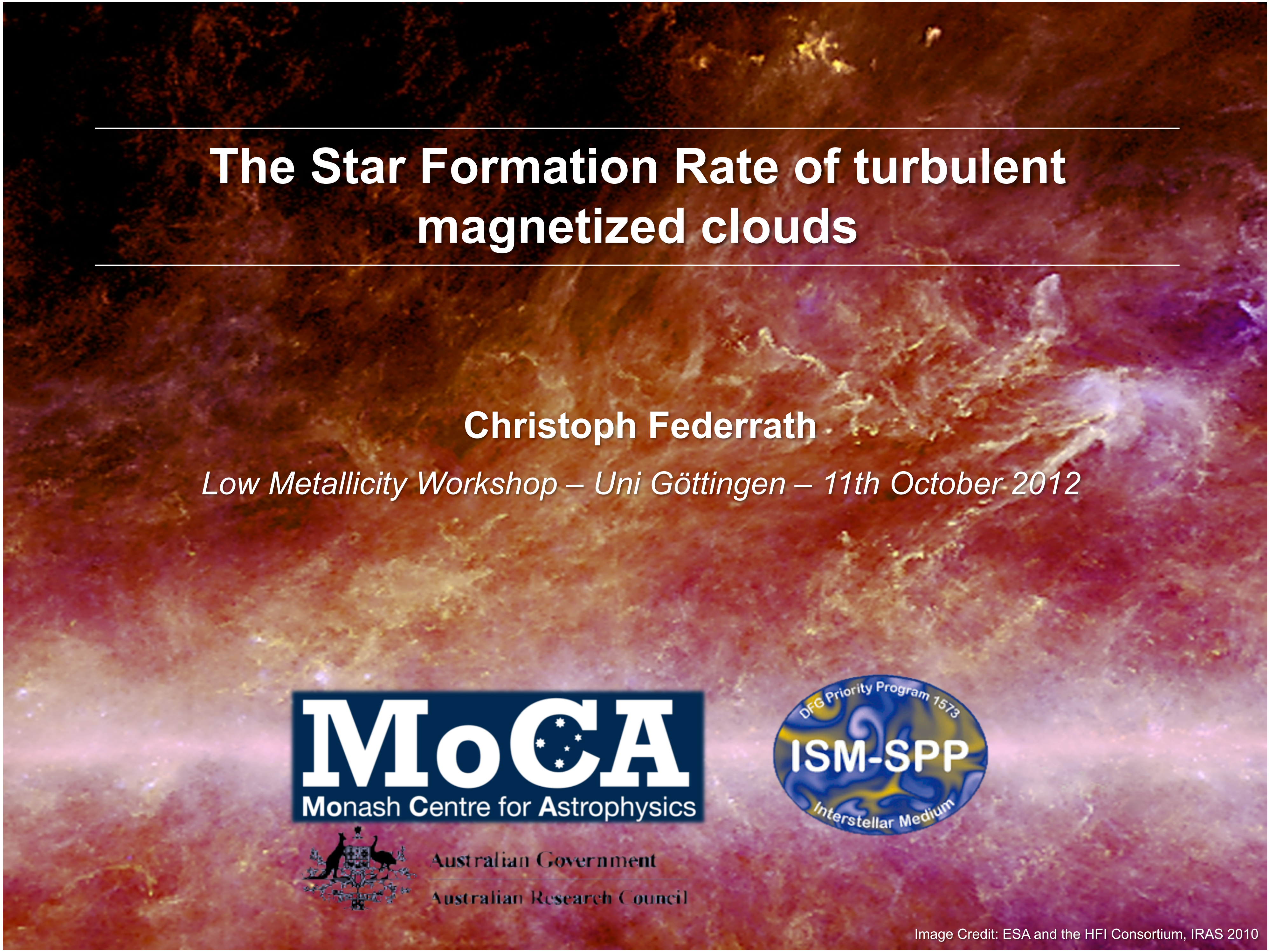


The Star Formation Rate of turbulent magnetized clouds

Christoph Federrath

Low Metallicity Workshop – Uni Göttingen – 11th October 2012

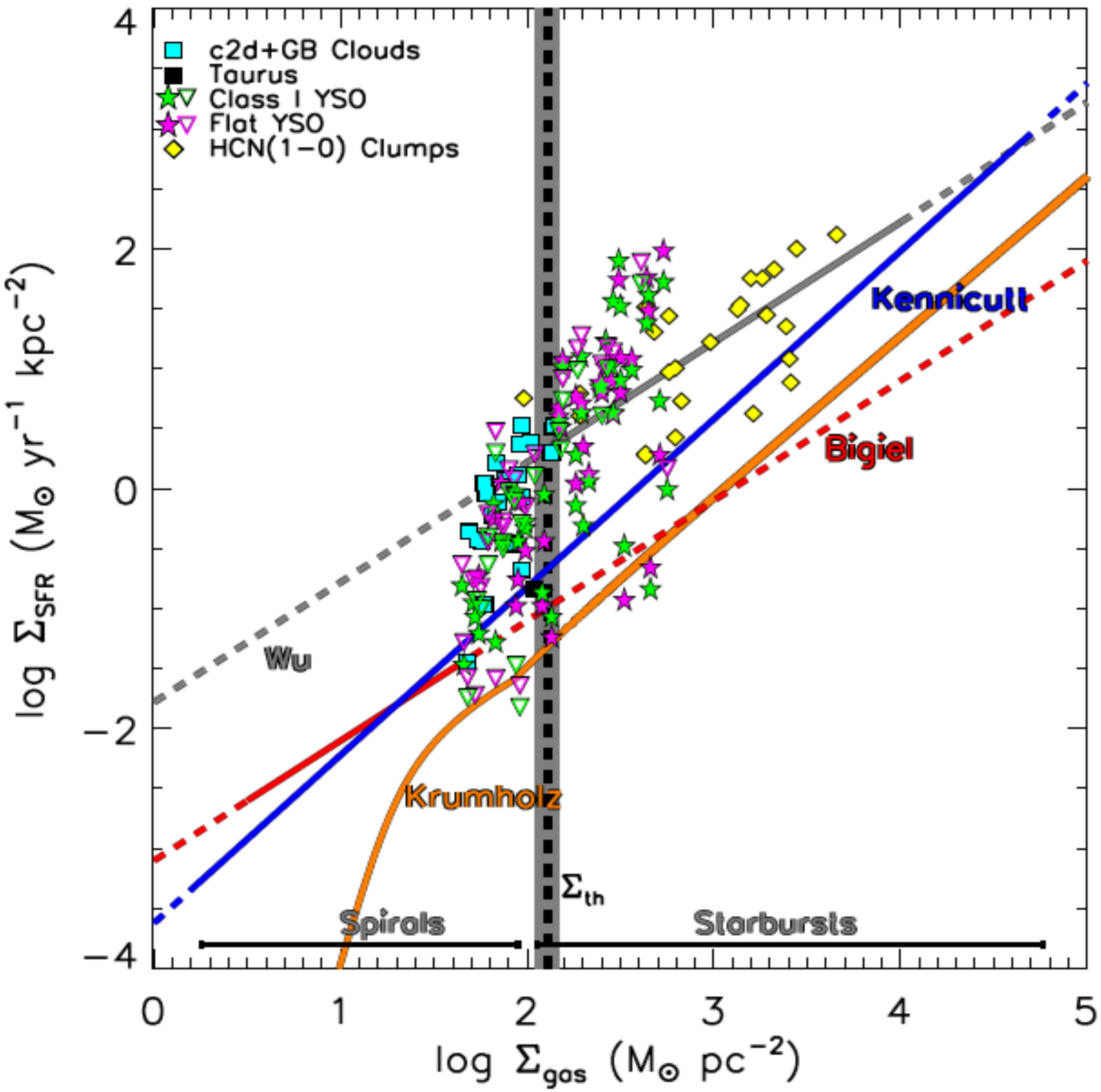
MoCA
Monash Centre for Astrophysics

Australian Government
Australian Research Council

DFG Priority Program 1573
ISM-SPP
Interstellar Medium

Image Credit: ESA and the HFI Consortium, IRAS 2010

Universal star formation “law”?



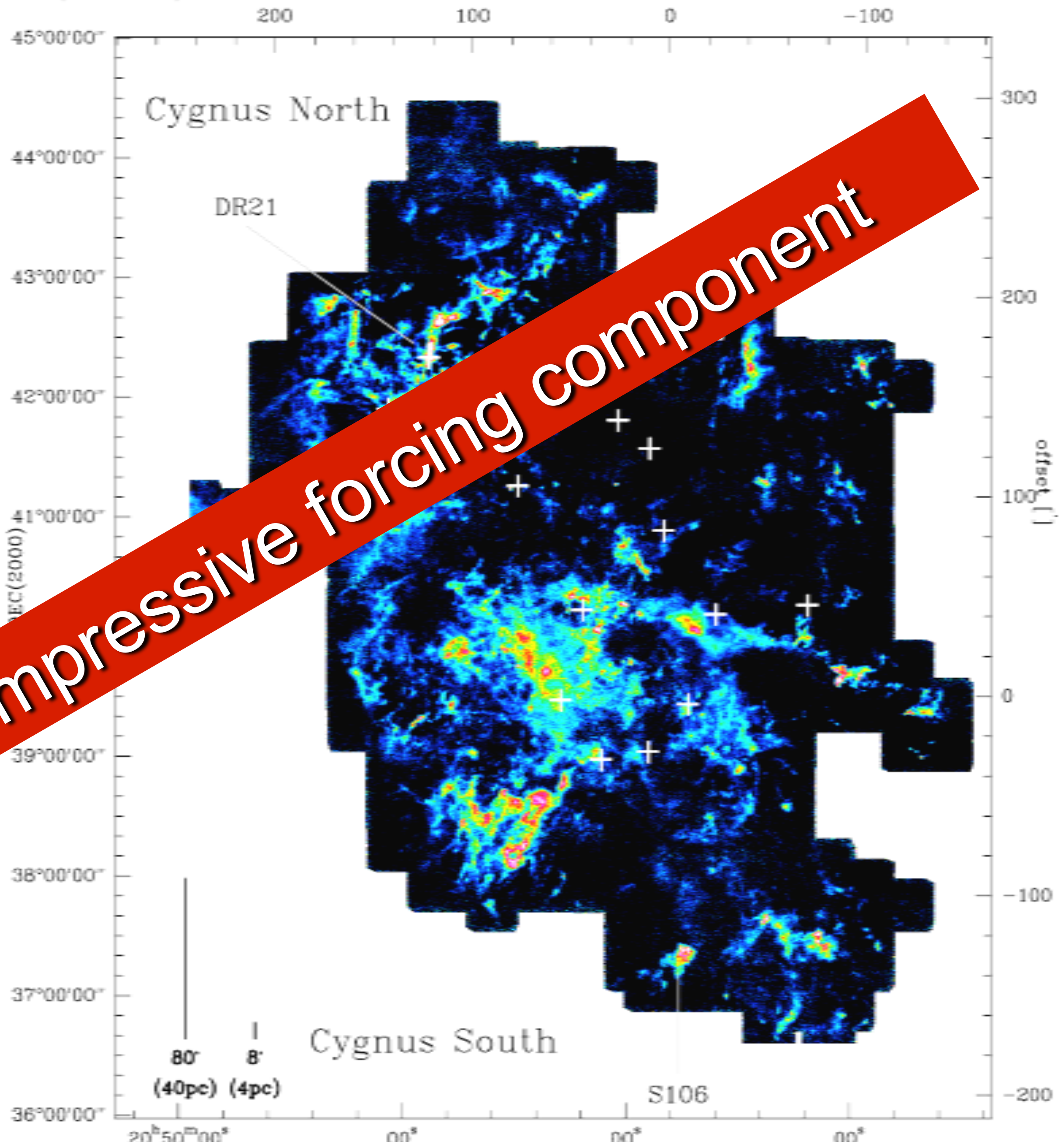
Galactic clouds (Heiderman+10; see also Lada+10)

Cygnus X: Schneider et al. (2011)

Giant molecular
cloud complex

Turbulence driven by

- **Supernova explosions?**
- **Ionization fronts?**
- **Protostellar jets/winds?**
- **MRI / shear?**
- **Gravitational infall?**
- **Galactic sprial shock?**



Significant compressive forcing component

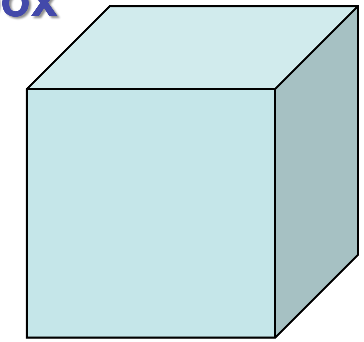
of thermal HII regions, the well-known star-forming regions DR21 and S106 are labelled

Turbulence forcing – solenoidal versus compressive

Typical setup for forced turbulence simulations:

e.g., Vazquez 1994, Padoan+1997, Passot+1998, Stone+1998, Mac Low 1999, Klessen+2000, Ostriker+2001, Heitsch+2001, Cho+2002, Boldyrev+2002, Li+2003, Haugen+2004, Padoan +2004, Jappsen+2005, Ballesteros+2006, Mee+Brandenburg 2006, Kritsuk+2007, Dib+2008, Offner+2008, Kowal+2008, Schmidt+2009, Cho+2009, Lemaster+2009, Glover+2010, Burkhart +2010, Price+2011, DelSordo+2011, Collins+2012, Walch+2012, Scannapieco+2012, Pan+2012, Robertson+2012, +++

"Turbulence in a box"



- ○ 3D, periodic boundary conditions
- ○ Isothermal gas: $P = c_s^2 \rho$
- ○ Driven to supersonic speeds (Mach 2 - 50)
- ○ Large-scale **Forcing Term f**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v} = -\frac{1}{\rho}\nabla P \cancel{- \nabla \Phi} + \boldsymbol{f}$$

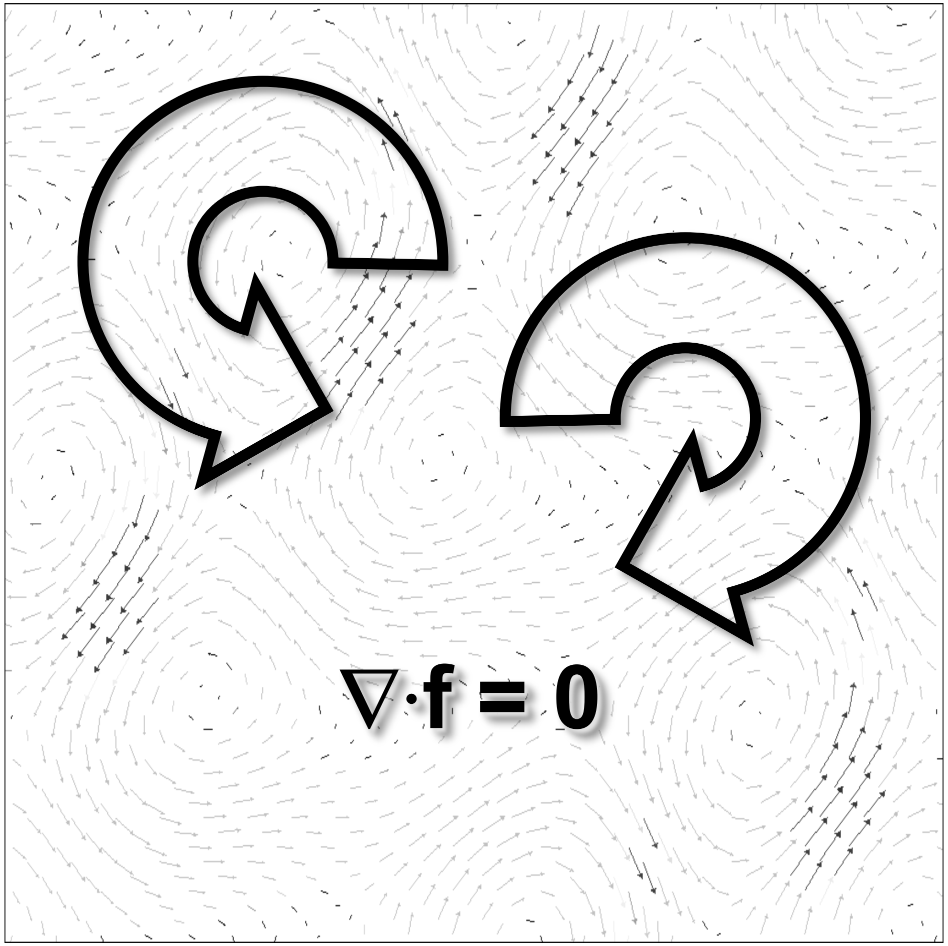
$$\cancel{\frac{\partial}{\partial t}(\rho e) + \nabla \cdot [\boldsymbol{v}(\rho e + P)] = \rho \boldsymbol{v} \nabla \Phi + \rho \boldsymbol{v} \cdot \boldsymbol{f}}$$

$$\cancel{\Delta \Phi = 4\pi G \rho}$$

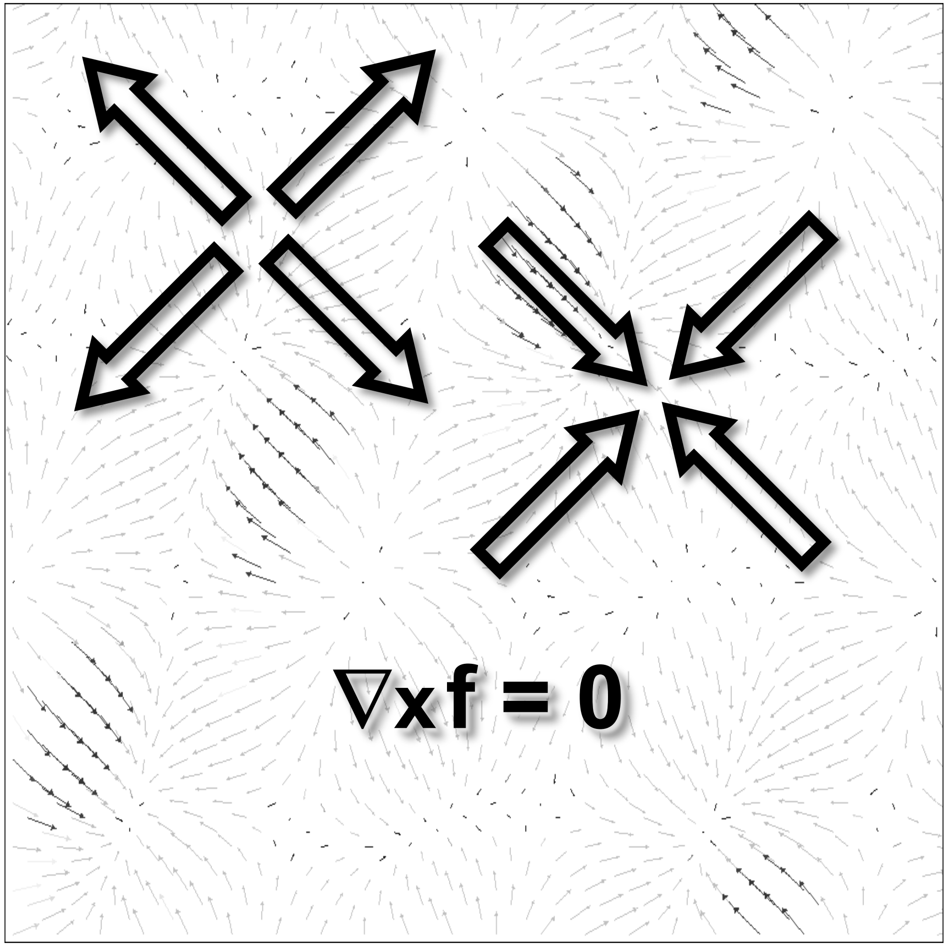
Turbulence forcing – solenoidal versus compressive

Ornstein-Uhlenbeck process (stochastic process with autocorrelation time)
→ **forcing varies smoothly in space and time,**
following a well defined random process

Solenoidal forcing

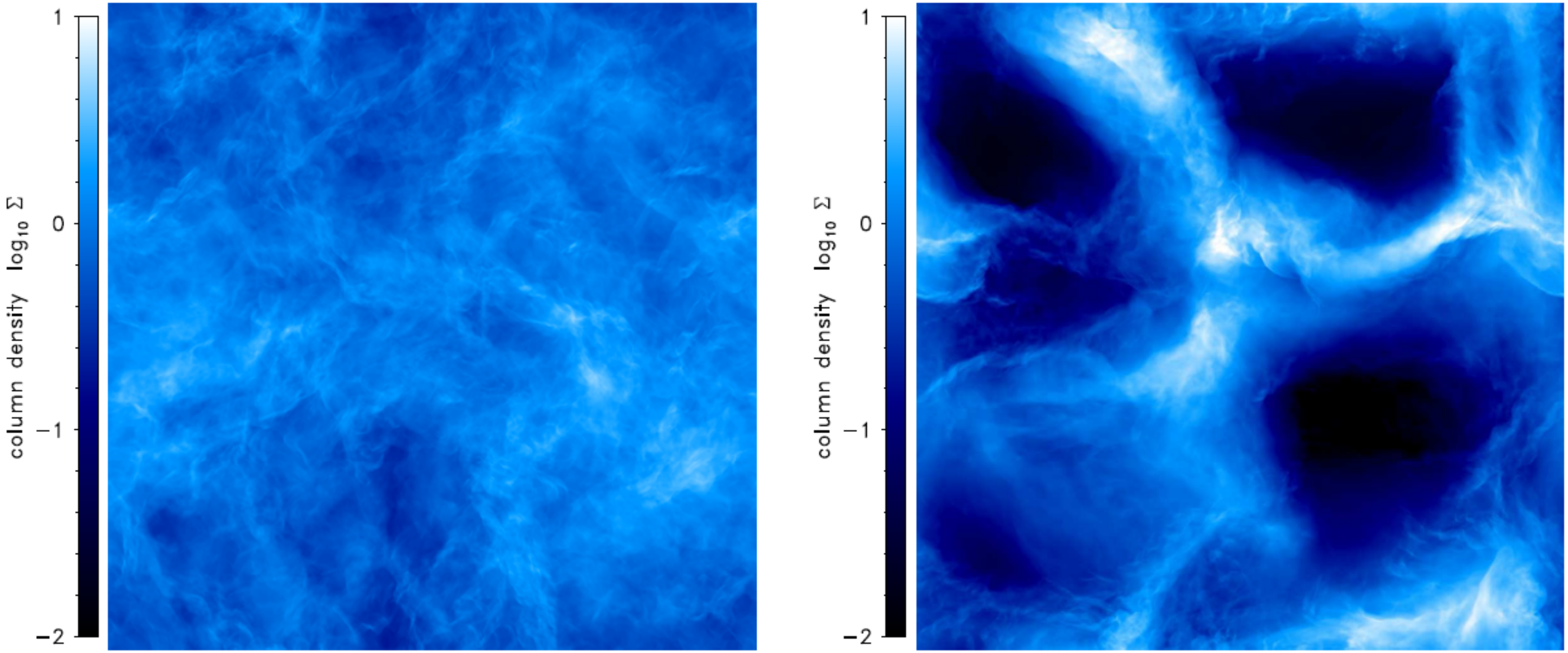


Compressive forcing



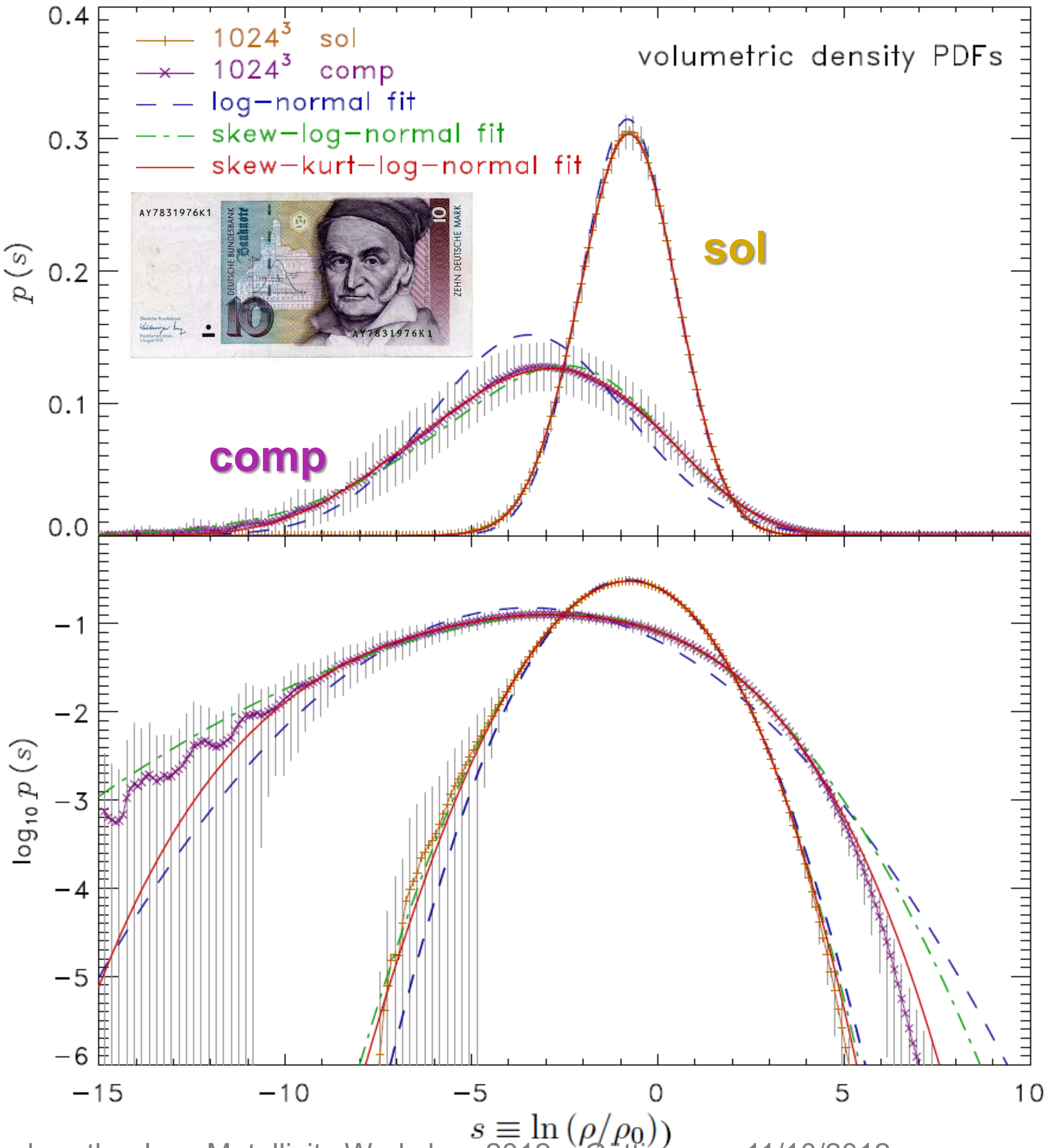
Turbulence forcing – solenoidal versus compressive

Solenoidal forcing **Compressive forcing**



Compressive forcing yields 3 times larger density dispersion for the same Mach number

The density PDF



gas density PDF

PDFs are close to log-normals:

$$p_s\,\mathrm{d}s = \frac{1}{\sqrt{2\pi\sigma_s^2}}\exp\left[-\frac{(s-\langle s\rangle)^2}{2\sigma_s^2}\right]\mathrm{d}s$$

$$s \equiv \ln(\rho/\rho_0)$$

Vazquez-Semadeni (1994)

$$\boxed{\sigma_s^2 = \ln\left(1 + b^2\mathcal{M}^2\right)}$$

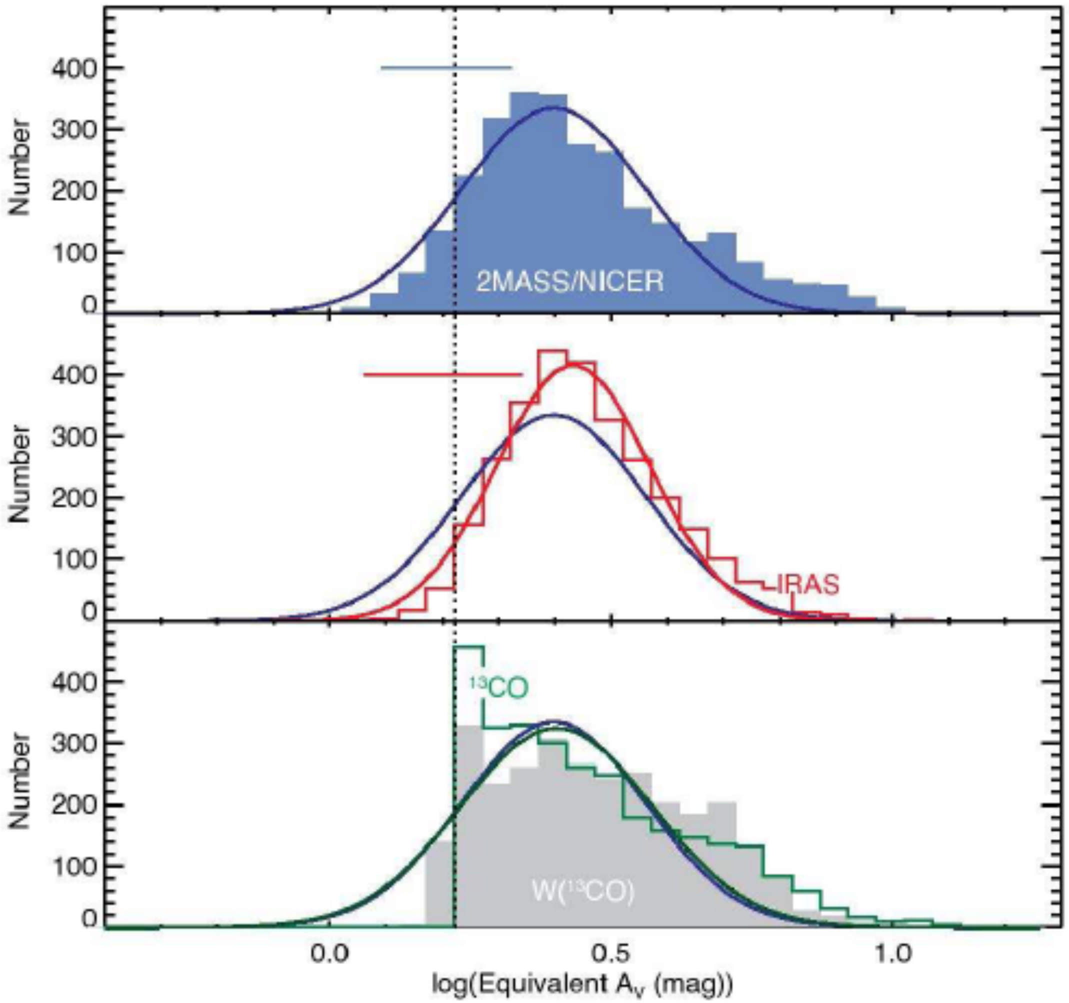
b = 1/3 (sol)

b = 1 (comp)

Federrath+08,10; Price+11, Konstandin+12

The density PDF

Comparison with observations (Perseus MC): Goodman, Pineda, & Schnee (2009)



dust extinction

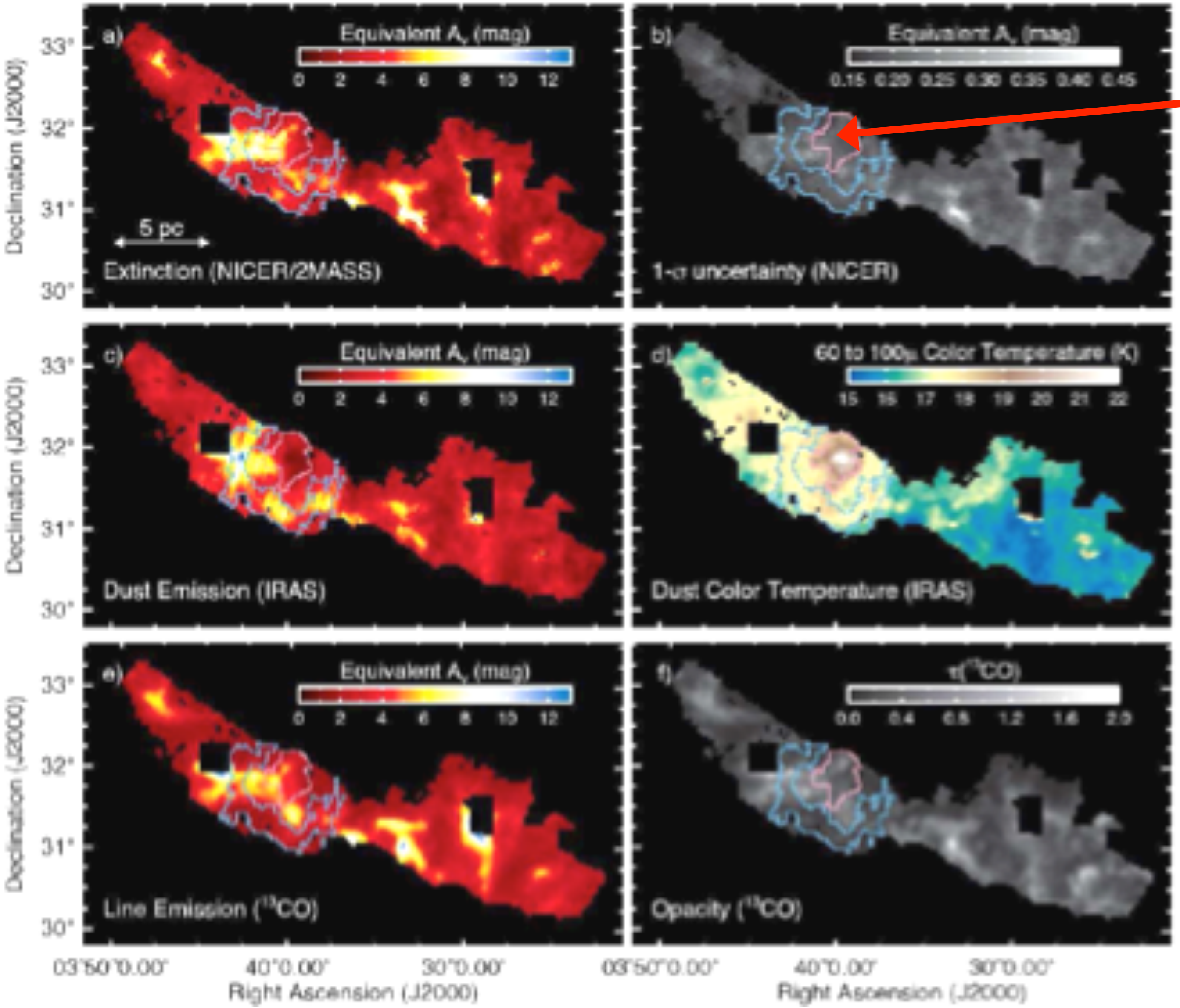
dust emission

^{13}CO line emission

→ **Column density PDFs are near log-normal distributions**

The density PDF

Comparison with observations: Goodman, Pineda, & Schnee (2009)



Expanding shell around massive B star

↓

Dominance of compressive modes expected

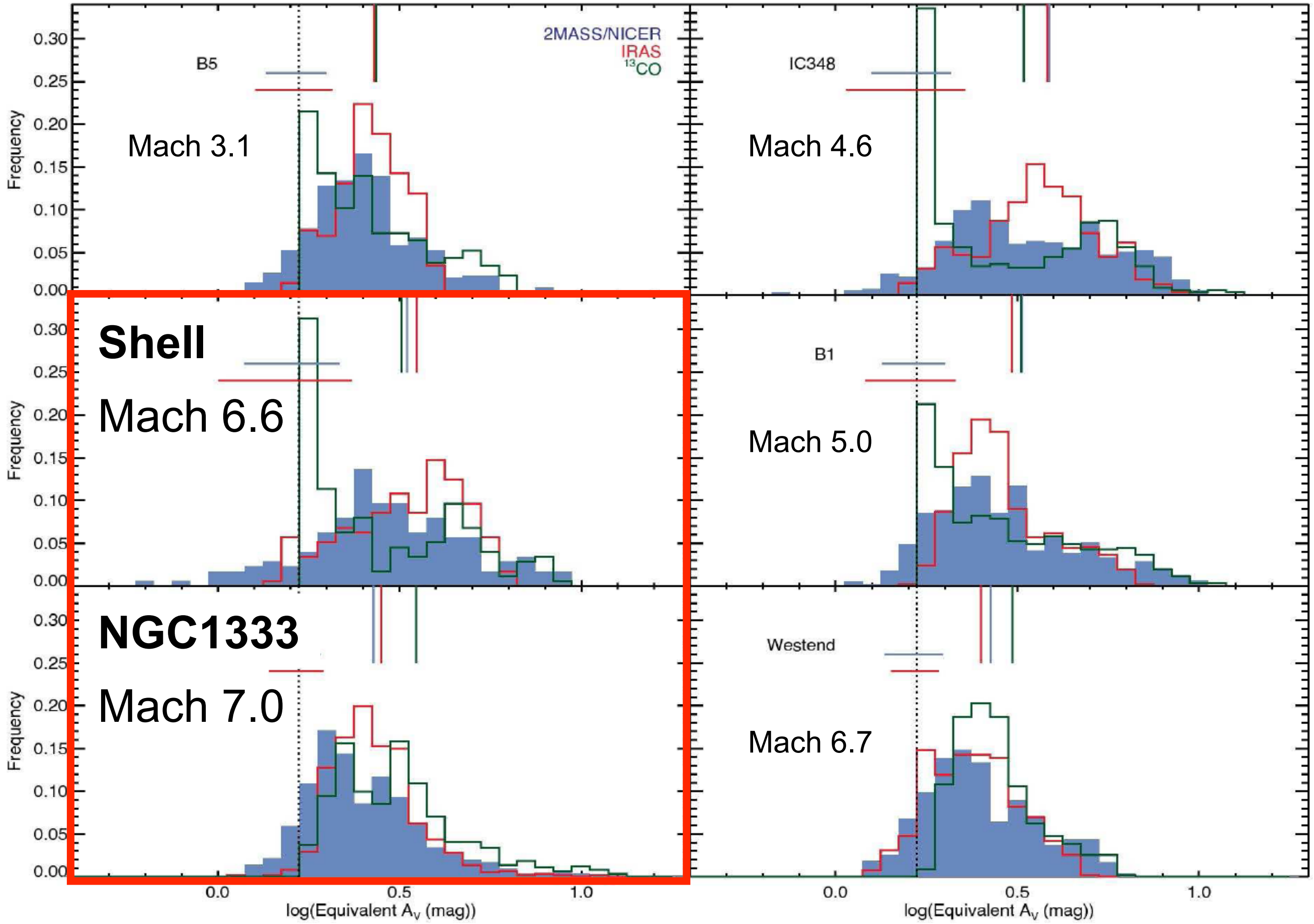
↓

Largest density dispersion in all of Perseus

Perseus MC

The density PDF

Comparison with observations: Goodman, Pineda, & Schnee 2009

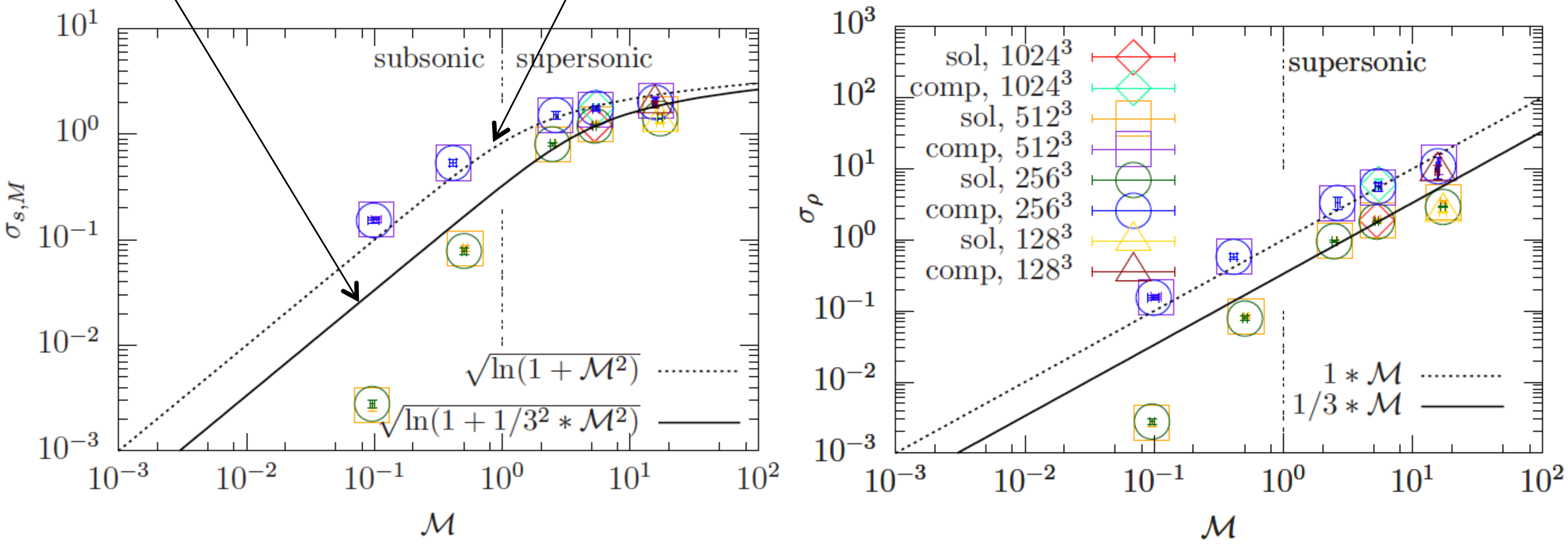


Density variance depends on Mach number AND forcing:

$$\sigma_s^2 = \ln\left(1 + \boxed{b^2}\mathcal{M}^2\right)$$

The density PDF

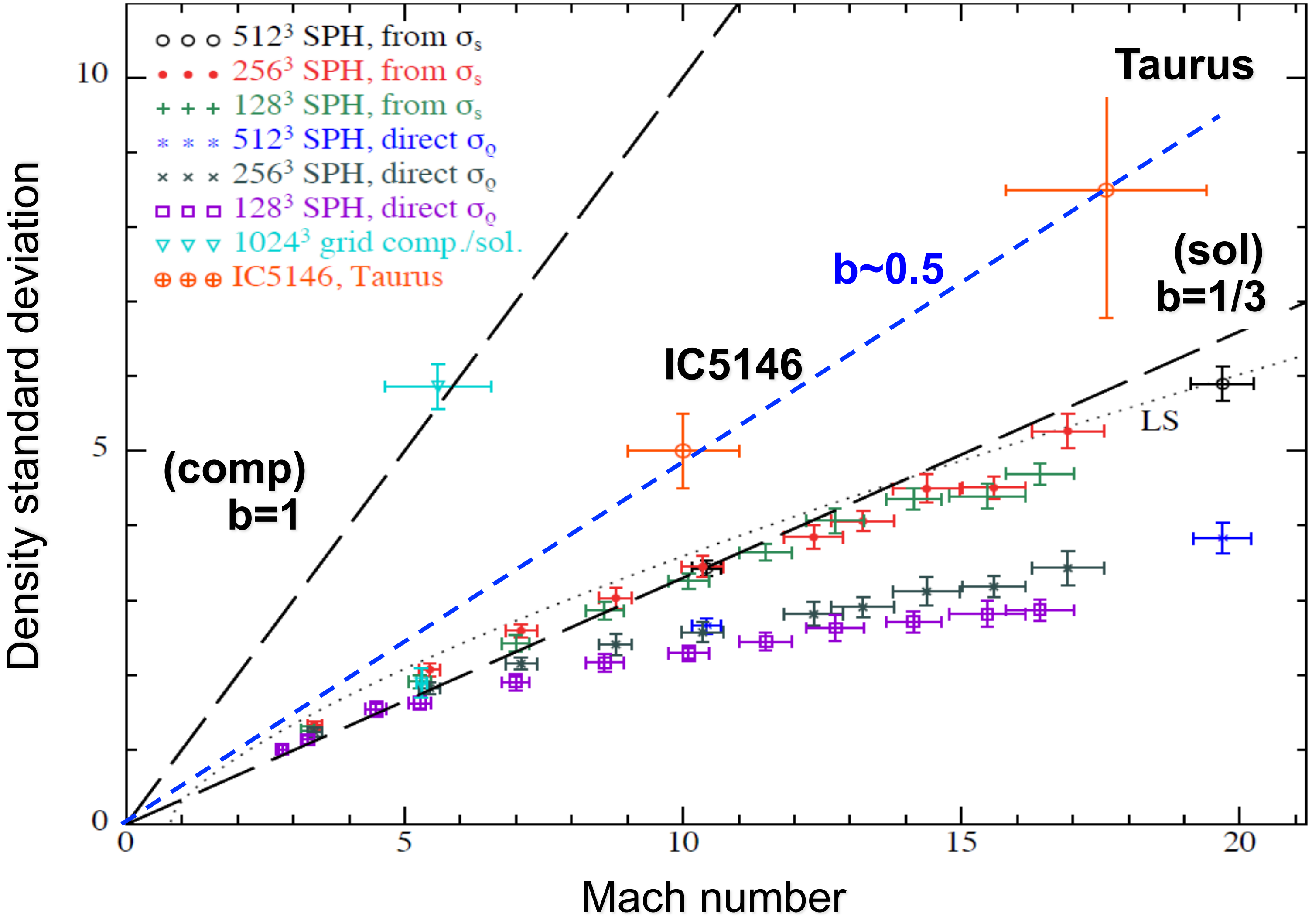
solenoidal (b=1/3) **compressive forcing (b=1)**



$$\boxed{\sigma_s^2 = \ln\left(1 + b^2\mathcal{M}^2\right)} \xrightarrow{p(s)} \boxed{\sigma_{\rho/\rho_0} = b\,\mathcal{M}}$$

Konstandin et al. (2012)

The density PDF



$$\boxed{\sigma_s^2 = \ln\left(1 + b^2 \mathcal{M}^2\right)} \xrightarrow{p(s)} \boxed{\sigma_{\rho/\rho_0} = b\,\mathcal{M}}$$

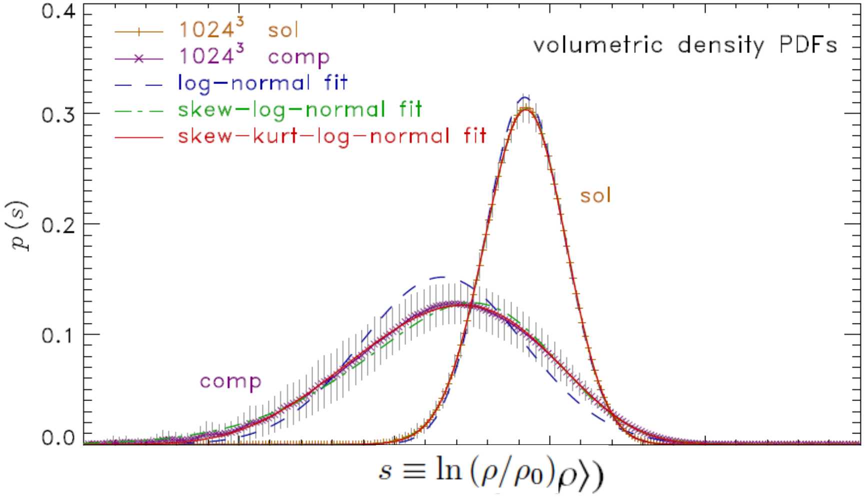
Price et al. (2011)

Compressive forcing and/or gravity required to explain observations

Density PDF → The Star Formation Rate

The density PDF

Density PDF is key for star formation theories



- **Initial Mass Function** (Padoan & Nordlund 02, Hennebelle & Chabrier 08,09, Elmegreen 11, Veltchev+12, Hopkins 12)
- **Star Formation Efficiency** (Elmegreen 08)
- **Kennicutt-Schmidt relation** (Elmegreen 02, Krumholz & McKee 05, Tassis 07)
- **Star Formation Rate** (Krumholz & McKee 05, Padoan & Nordlund 11)

All based on integrals over the turbulent density PDF

$$\mathrm{SFR}_{\mathrm{ff}} = \frac{\epsilon_{\mathrm{core}}}{\phi_t} \int_{x_{\mathrm{crit}}}^{\infty} x p(x)\, dx$$

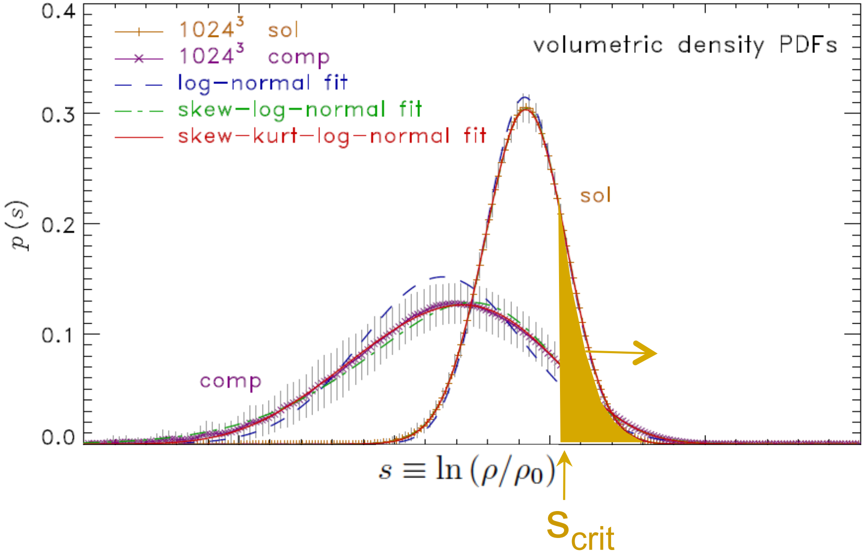
Krumholz & McKee (2005), Padoan & Nordlund 2011; Hennebelle & Chabrier (2011)

The Star Formation Rate

Statistical Theory for the Star Formation Rate:

SFR ~ Mass/time

$$\mathrm{SFR}_{\mathrm{ff}} = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \overbrace{\frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)}}^{\textbf{freefall time}} \overbrace{\frac{\rho}{\rho_0}}^{\textbf{mass fraction}} p(s)\,\mathrm{d}s$$



Hennebelle & Chabrier (2011) : “multi-freefall model”

see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate

Statistical Theory for the Star Formation Rate:

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s-s_0)^2}{2\sigma_s^2}\right)$$

$$s = \ln(\rho/\rho_0) \qquad t_{\mathrm{ff}}(\rho) = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

$$\begin{aligned}
\mathrm{SFR}_{\mathrm{ff}} &= \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \overbrace{\frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)}}^{\text{freefall time}} \overbrace{\frac{\rho}{\rho_0}}^{\text{mass fraction}} p(s)\,\mathrm{d}s \overset{\downarrow}{=} \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \exp\left(\frac{3}{2}s\right) p(s)\,\mathrm{d}s \\
&= \frac{\epsilon}{2} \exp\left(\frac{3}{8}\sigma_s^2\right) \left[1 + \mathrm{erf}\left(\frac{\sigma_s^2 - s_{\mathrm{crit}}}{\sqrt{2\sigma_s^2}}\right)\right]
\end{aligned}$$

Hennebelle & Chabrier (2011) : “multi-freefall model”

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The Star Formation Rate

Statistical Theory for the Star Formation Rate:

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s-s_0)^2}{2\sigma_s^2}\right)$$

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SFR ~ Mass/time

$$\mathrm{SFR}_{\mathrm{ff}} = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \overbrace{\frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)}}^{\text{freefall time}} \overbrace{\frac{\rho}{\rho_0}}^{\text{mass fraction}} p(s)\,\mathrm{d}s = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \exp\left(\frac{3}{2}s\right) p(s)\,\mathrm{d}s$$

$$= \frac{\epsilon}{2} \exp\left(\frac{3}{8}\sigma_s^2\right)\left[1 + \mathrm{erf}\left(\frac{\sigma_s^2 - s_{\mathrm{crit}}}{\sqrt{2\sigma_s^2}}\right)\right]$$

Hennebelle & Chabrier (2011) : “multi-freefall model”

$$\boxed{s_{\mathrm{crit}} \propto \ln\left(\alpha_{\mathrm{vir}}\,\mathcal{M}^2\right)}$$

(KM05, PN11)

$$\boxed{\sigma_s^2 = \ln\left(1 + b^2\mathcal{M}^2\right)}$$

(PN97, PV98, F08,10; Price+11)

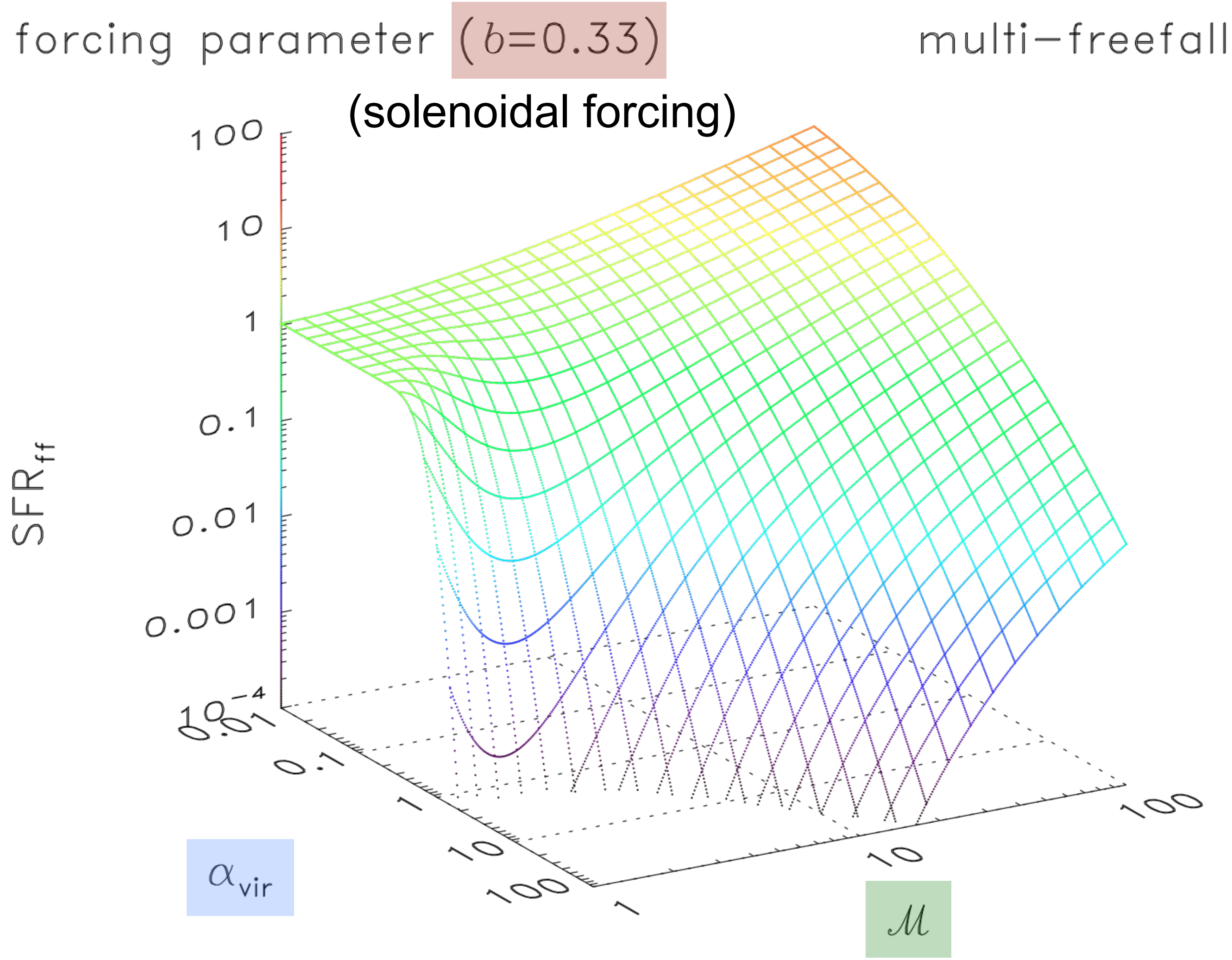
$$\mathrm{SFR}_{\mathrm{ff}} = \mathrm{SFR}_{\mathrm{ff}}\left(\underbrace{\alpha_{\mathrm{vir}}}_{2E_{\mathrm{kin}}/E_{\mathrm{grav}}}, \underbrace{b}_{\text{forcing}}, \underbrace{\mathcal{M}}_{\text{Mach number}}\right)$$

see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate

$$\mathrm{SFR}_{\mathrm{ff}} = \mathrm{SFR}_{\mathrm{ff}}\,(\alpha_{\mathrm{vir}}, b, \mathcal{M})$$

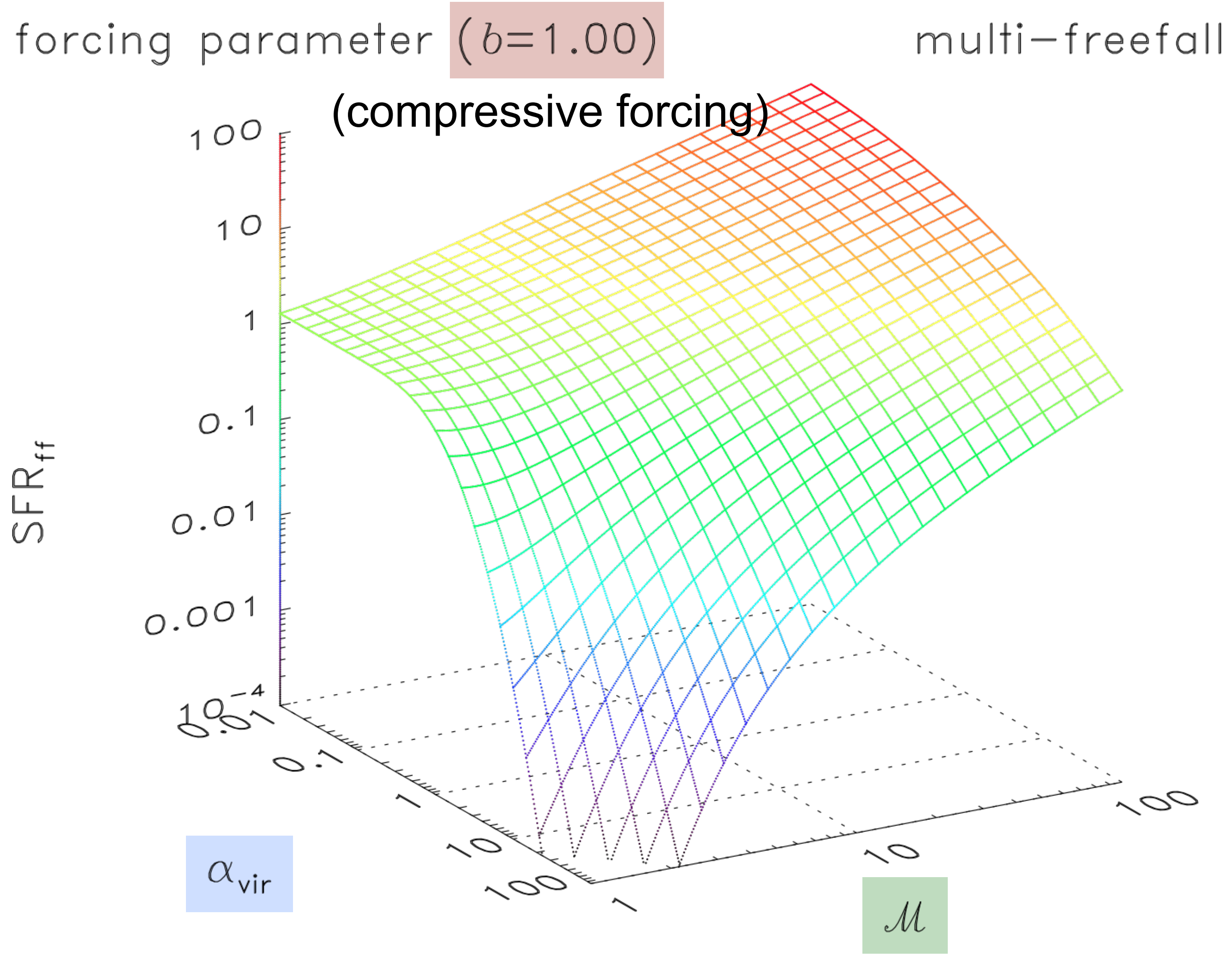
$2\,E_{\mathrm{kin}}/E_{\mathrm{grav}}$ **forcing** **Mach number**



see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate

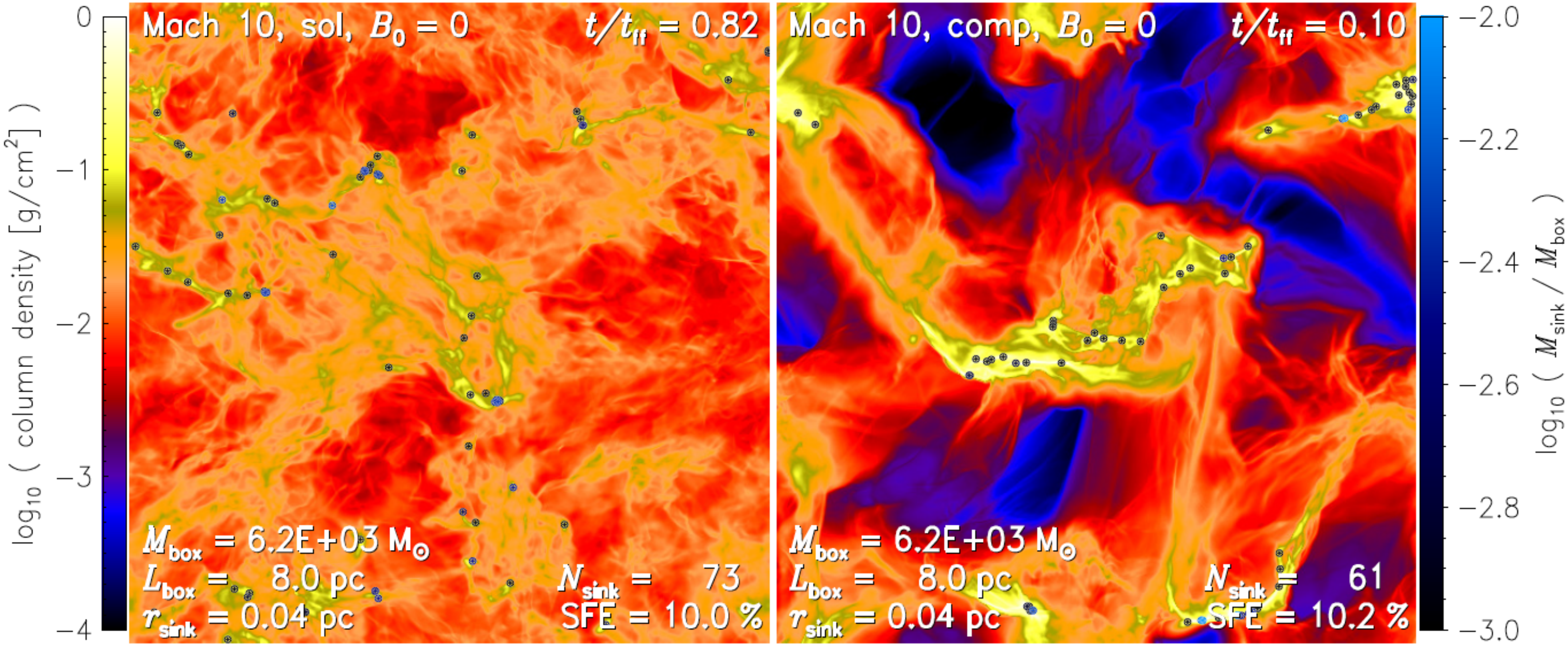
$\mathrm{SFR_{ff}} = \mathrm{SFR_{ff}}\,(\alpha_{\mathrm{vir}}, b, \mathcal{M})$ $2\,E_{\mathrm{kin}}/E_{\mathrm{grav}}$ **forcing** **Mach number**



see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate

Numerical Test at Mach 10



Solenoidal Forcing

Compressive Forcing

SFR$_{ff}$ (simulation) = **0.14** x **20** SFR$_{ff}$ (simulation) = **2.8**

SFR$_{ff}$ (theory) = **0.15** x **15** SFR$_{ff}$ (theory) = **2.3**

Theory and Simulation agree well.

see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate – Magnetic fields

Statistical Theory for the Star Formation Rate:

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s-s_0)^2}{2\sigma_s^2}\right)$$

$$s = \ln(\rho/\rho_0) \qquad t_{\mathrm{ff}}(\rho) = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

SFR ~ Mass/time

$$\mathrm{SFR}_{\mathrm{ff}} = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \overbrace{\frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)}}^{\textbf{freefall time}} \overbrace{\frac{\rho}{\rho_0}}^{\textbf{mass fraction}} p(s)\,\mathrm{d}s = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \exp\left(\frac{3}{2}s\right) p(s)\,\mathrm{d}s$$

$$= \frac{\epsilon}{2} \exp\left(\frac{3}{8}\sigma_s^2\right)\left[1 + \mathrm{erf}\left(\frac{\sigma_s^2 - s_{\mathrm{crit}}}{\sqrt{2\sigma_s^2}}\right)\right]$$

MAGNETIC FIELD:

$$P_{\mathrm{th}} \rightarrow P_{\mathrm{th}} + P_{\mathrm{mag}} \qquad \mathcal{M} \rightarrow \mathcal{M}\left(1+\beta^{-1}\right)^{-1/2}$$

$$s_{\mathrm{crit}} \propto \ln\left(\alpha_{\mathrm{vir}}\,\mathcal{M}^2 \frac{\beta}{\beta+1}\right.$$

$$\sigma_s^2 = \ln\left(1 + b^2 \mathcal{M}^2 \frac{\beta}{\beta+1}\right.$$

(PN11; Molina+2012)

$$\mathrm{SFR}_{\mathrm{ff}} = \mathrm{SFR}_{\mathrm{ff}}\left(\alpha_{\mathrm{vir}}, b, \mathcal{M}, \beta\right)$$

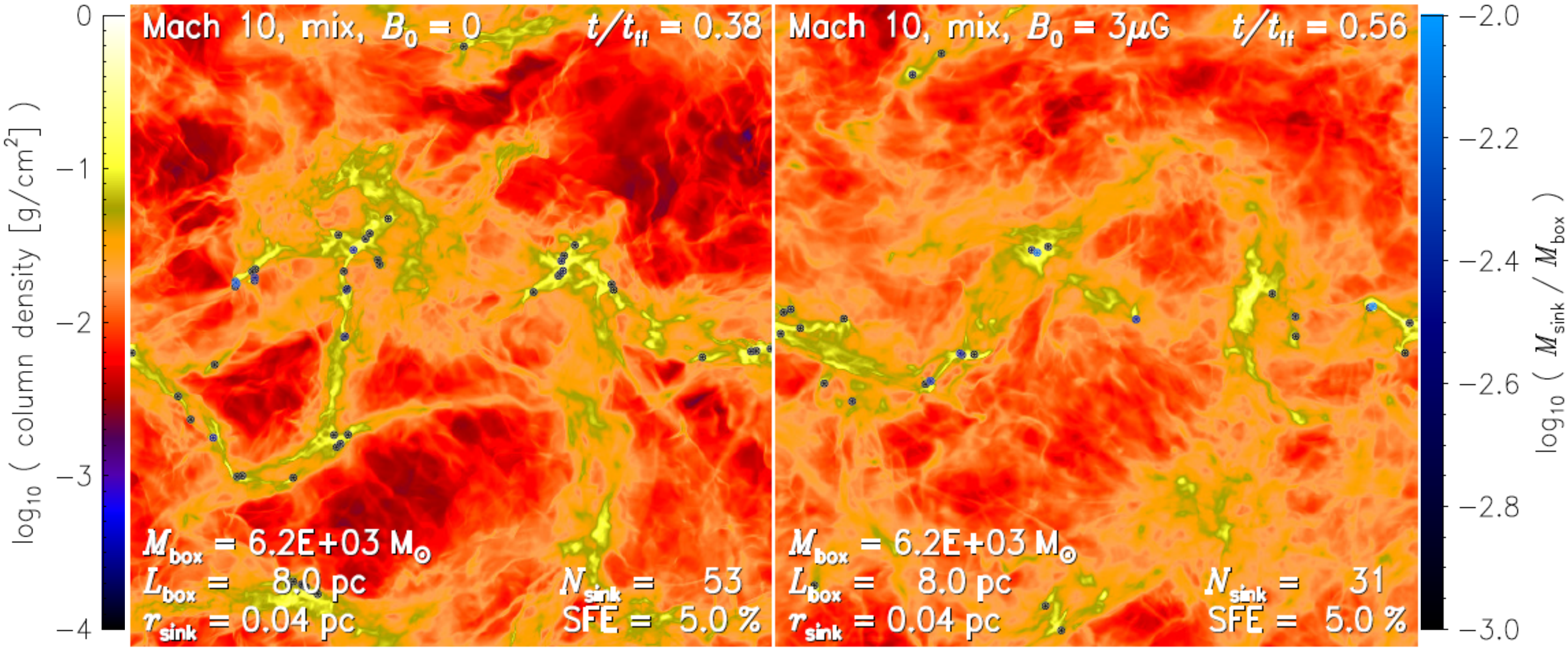
$2\,E_{\mathrm{kin}}/E_{\mathrm{grav}}$ forcing Mach number plasma $\beta = P_{\mathrm{th}}/P_{\mathrm{mag}}$

see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate – Magnetic fields

Numerical Test at Mach 10 with mixed forcing

B=0 (M_A=∞, β = ∞) B=3μG (M_A=2.7, β = 0.2)



SFR_{ff} (simulation) = **0.46**	x **0.63**	SFR_{ff} (simulation) = **0.29**
SFR_{ff} (theory) = **0.45**	x **0.40**	SFR_{ff} (theory) = **0.18**

Magnetic field reduces SFR and fragmentation (by factor ~2).

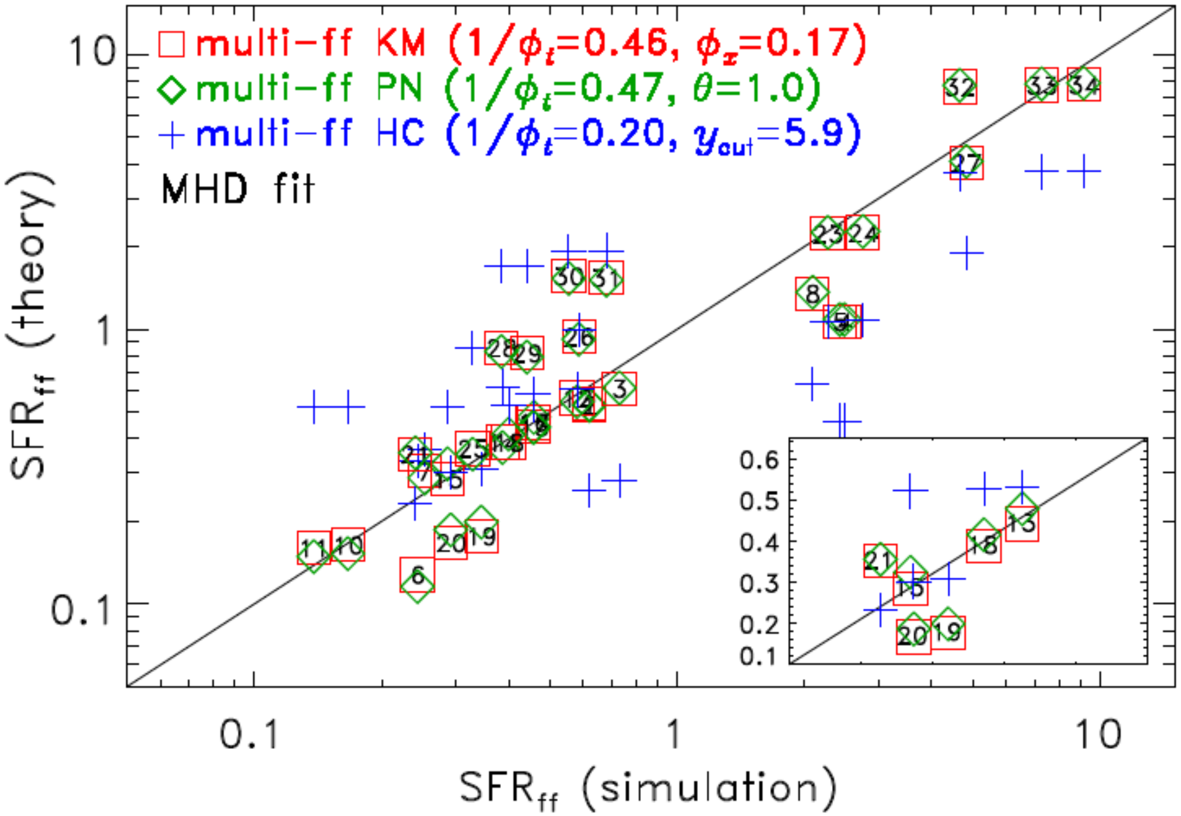
see Federrath & Klessen 2012, ApJ accepted

The Star Formation Rate of MHD turbulence

Compare simulations with

- cloud masses of 300 – $4\times10^6\ M_\odot$
- solenoidal, mixed, and compressive forcing
- sonic Mach numbers 3 – 50
- Alfvén Mach numbers 1 – infinity

Simulations *vs.* Theory



see Federrath & Klessen 2012, ApJ accepted

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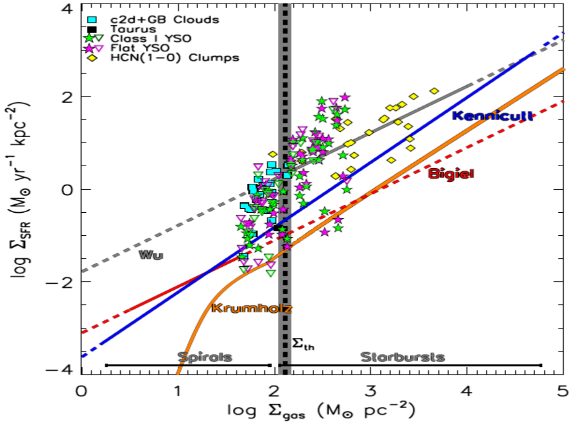
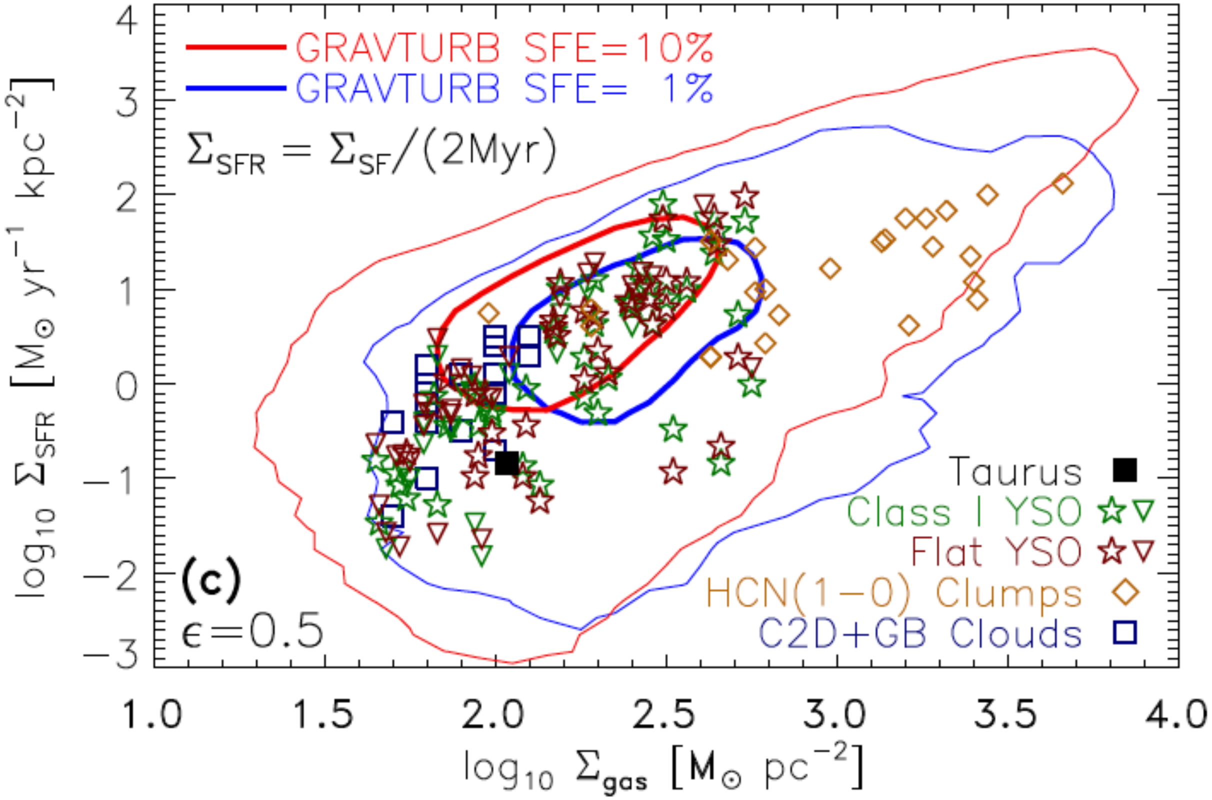


(Heiderman et al. 2010)

Simulations *vs.* Observations



see Federrath & Klessen 2012, ApJ accepted

Conclusions

Star Formation Rate (SFR) from supersonic, magnetized turbulence:

- MHD turbulence is key for star formation (see Federrath & Klessen 2012, ApJ)
- $\mathrm{SFR}_{\text{(compressive forcing)}} > 10 \times \mathrm{SFR}_{\text{(solenoidal forcing)}}$
- SFR as integral over density distribution (PDF) depends on
  - virial parameter
  - turbulent forcing parameter
  - sonic Mach number
  - plasma beta
- Magnetic fields reduce SFR, consistent with theoretical model prediction
- Good agreement between theory, simulations and observations